

【SAP2000与结构力学】系列课程

# SAP2000 拱结构分析

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## 拱桥

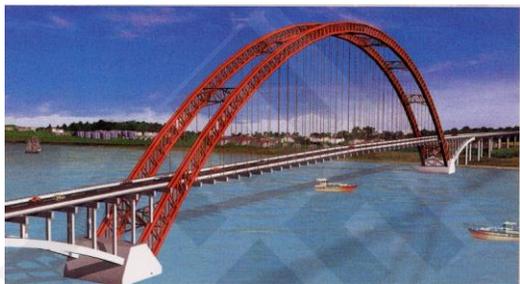


下承式拱桥



上承式拱桥（赵州桥）

## 拱桥



中承式拱桥（钢桥）



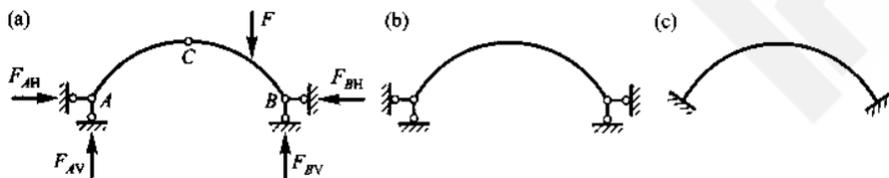
中承式拱桥（混凝土桥）

筑信达

## 拱结构的类型

### ❖ 简单体系

- 三铰拱：静定结构，对地基条件适应性强。
- 两铰拱：一次超静定结构，介于其余两者之间。
- 无铰拱：三次超静定结构，弯矩分布均匀，构造简单。

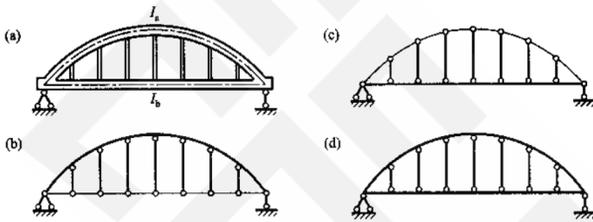


筑信达

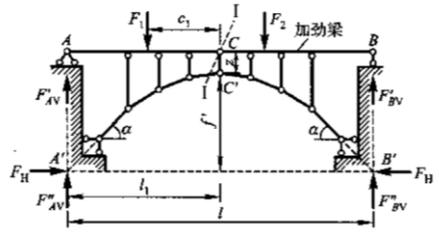
# 拱结构的类型

## 组合体系

- 无推力拱：系杆拱...
- 有推力拱：梁+拱



系杆拱（杆和拱的相对刚度）



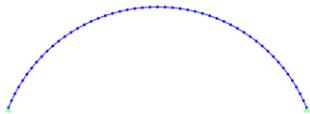
拱式组合结构

筑信达

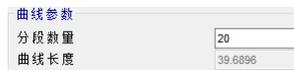
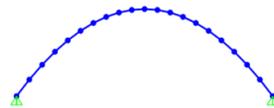
# SAP2000 曲线框架

## “以直代曲”

- 圆弧线：第三点或圆心和半径



- 抛物线：第三点



筑信达

# SAP2000 曲线框架

## ❖ “以直代曲”

- 悬链线：交互式数据库编辑 → Excel (函数) → 交互式数据库编辑



交互式数据库编辑: Joint Coordinates

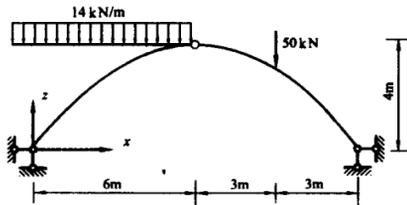
Joint	CoordSys	CoordType	XorR	Y	Z	SpecialR
1	GLOBAL	Cartesian	6.661E-17	77.40000	0	0.15 No
2	GLOBAL	Cartesian	0.06	77.40000	0	0.15 No
3	GLOBAL	Cartesian	0.06	77.34000	0	0.14489 No
4	GLOBAL	Cartesian	9.992E-17	77.28000	0	0.1299 No
5	GLOBAL	Cartesian	0.06	77.22000	0	0.1207 No
6	GLOBAL	Cartesian	0	77.22000	0	0.1207 No
7	GLOBAL	Cartesian	0.06	77.22000	0	0.1207 No
8	GLOBAL	Cartesian	6.661E-17	77.16000	0	0.1057 No
9	GLOBAL	Cartesian	0.06	77.16000	0	0.1057 No
10	GLOBAL	Cartesian	9.992E-17	77.10000	0	0.1007 No
11	GLOBAL	Cartesian	0.06	77.10000	0	0.1007 No
12	GLOBAL	Cartesian	6.661E-17	77.04000	0	0.0857 No
13	GLOBAL	Cartesian	0.06	77.04000	0	0.0857 No
14	GLOBAL	Cartesian	6.661E-17	76.98000	0	0.0707 No
15	GLOBAL	Cartesian	0.06	76.98000	0	0.0707 No
16	GLOBAL	Cartesian	9.992E-17	76.92000	0	0.0557 No
17	GLOBAL	Cartesian	0.06	76.92000	0	0.0557 No
18	GLOBAL	Cartesian	6.661E-17	76.86000	0	0.0407 No
19	GLOBAL	Cartesian	0.06	76.86000	0	0.0407 No
20	GLOBAL	Cartesian	6.661E-17	76.80000	0	0.0257 No
21	GLOBAL	Cartesian	0.06	76.80000	0	0.0257 No
22	GLOBAL	Cartesian	6.661E-17	76.74000	0	0.0107 No
23	GLOBAL	Cartesian	0.06	76.74000	0	0.0107 No
24	GLOBAL	Cartesian	6.661E-17	76.68000	0	0.0057 No
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筑信达

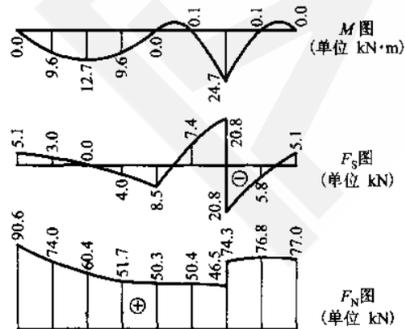
# 手算 vs 电算

## ❖ 拱轴线为抛物线的三铰拱

- 拱顶铰：端部释放
- 线荷载的作用方向
- 集中荷载的作用位置



三铰拱



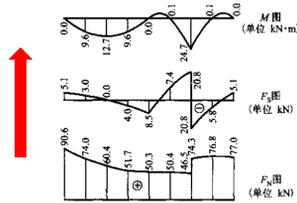
筑信达

# 手算vs电算

## ❖ 三铰拱“以直代曲”逐步细化

- 如何细化？
- 支座反力变化？
- 拱内力变化？

P KN	V2 KN	M3 KN·m
-50.177	8.918	3.486E-12
-50.101	9.336	9.172E-13
-50.006	9.837	4.042E-13
-49.573	11.824	3.851E-15
-48.686	15.067	1.339E-14
-47.209	19.198	1.534E-14



增加单元数量，减小单元尺寸

筑信达

# 合理拱轴线

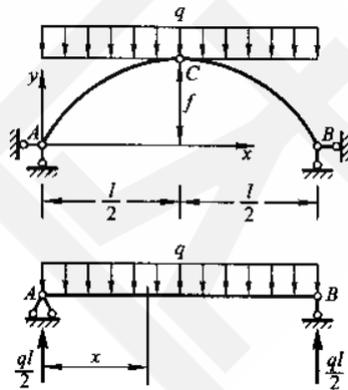
## ❖ 抛物线

- 支座反力：由荷载和三铰拱位置控制
- 内力：与拱轴线有关

合理拱轴线：全部截面的弯矩为零时对应的拱轴线。在**竖向荷载**作用下，纵坐标 $y$ 与相应简支梁的弯矩图成正比。

$$M = M^0 - F_H y = 0$$

$$y = \frac{M^0}{F_H}$$



抛物线

$$y = \frac{M^0}{F_H} = \frac{4f}{l^2} x(l-x)$$

筑信达

## 合理拱轴线

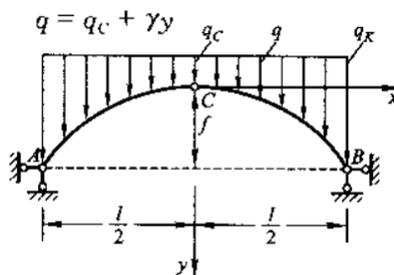
### ❖ 悬链线

- 支座反力：由荷载和三铰拱位置控制
- 内力：与拱轴线有关

合理拱轴线：全部截面的弯矩为零时对应的拱轴线。在**竖向荷载**作用下，纵坐标 $y$ 与相应简支梁的弯矩图成正比。

$$M = M^0 - F_H(f - y)$$

$$f - y = \frac{M^0}{F_H}$$



悬链线

$$y = \frac{f}{m-1} (\cosh K\xi - 1)$$

$$K = \ln(m + \sqrt{m^2 - 1})$$

$$m = \frac{q_K}{q_C}$$

筑信达

## 合理拱轴线

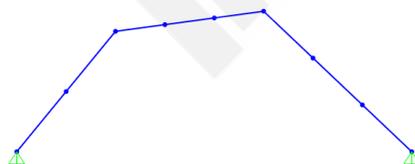
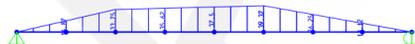
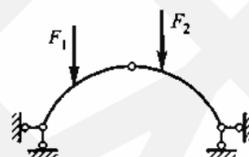
### ❖ 多折线

- 支座反力：由荷载和三铰拱位置控制
- 内力：与拱轴线有关

合理拱轴线：全部截面的弯矩为零时对应的拱轴线。在**竖向荷载**作用下，纵坐标 $y$ 与相应简支梁的弯矩图成正比。

$$M = M^0 - F_H y = 0$$

$$y = \frac{M^0}{F_H}$$



多折线

筑信达

## 合理拱轴线

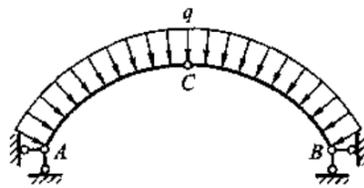
### 圆弧线

- 支座反力：由荷载和三铰拱位置控制
- 内力：与拱轴线有关

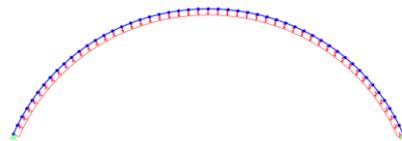
合理拱轴线是指：在均布荷载作用下，拱任一截面的弯矩为零时对应的拱轴线。与相应简支梁的弯矩图相比。

$$M = M^0 - F_H y = 0$$

$$y = \frac{M^0}{F_H}$$



圆弧线

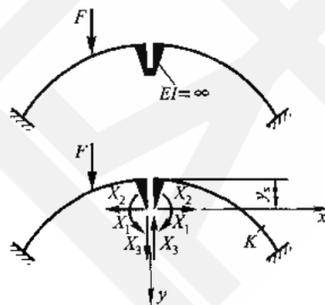
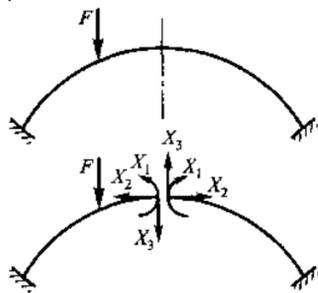


基于框架局部轴施加均布线荷载

筑信达

## 弹性中心法

### 无铰拱



$$\left. \begin{aligned} \delta_{11} X_1 + \delta_{12} X_2 + \delta_{13} X_3 + \Delta_{1P} &= 0 \\ \delta_{21} X_1 + \delta_{22} X_2 + \delta_{23} X_3 + \Delta_{2P} &= 0 \\ \delta_{31} X_1 + \delta_{32} X_2 + \delta_{33} X_3 + \Delta_{3P} &= 0 \end{aligned} \right\}$$

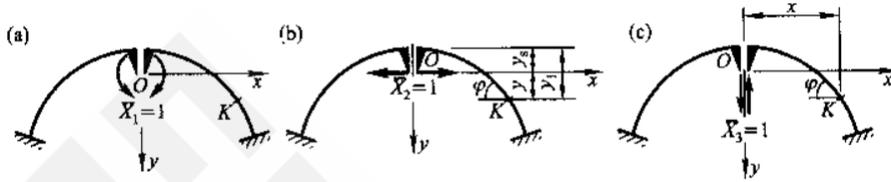


$$\begin{aligned} \delta_{11} X_1 + \Delta_{1P} &= 0 \\ \delta_{22} X_2 + \Delta_{2P} &= 0 \\ \delta_{33} X_3 + \Delta_{3P} &= 0 \end{aligned}$$

筑信达

## 弹性中心法

### ❖ 无铰拱



$$\begin{aligned} \delta_{12} = \delta_{21} &= \int \frac{\bar{M}_1 \bar{M}_2 ds}{EI} + \int \frac{\bar{F}_{N1} \bar{F}_{N2} ds}{EA} + \int k \frac{\bar{F}_{s1} \bar{F}_{s2} ds}{GA} \\ &= \int \frac{\bar{M}_1 \bar{M}_2 ds}{EI} + 0 + 0 \\ &= \int y \frac{ds}{EI} = \int (y_1 - y_2) \frac{ds}{EI} = \int y_1 \frac{ds}{EI} - y_2 \int \frac{ds}{EI} \end{aligned}$$

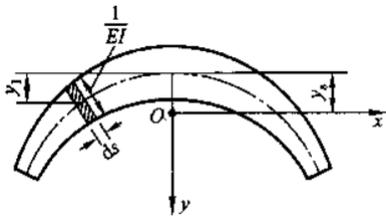
$$\begin{aligned} \delta_{13} = \delta_{31} &= 0 \\ \delta_{23} = \delta_{32} &= 0 \end{aligned}$$

$$\rightarrow y_s = \frac{\int y_1 \frac{ds}{EI}}{\int \frac{ds}{EI}}$$

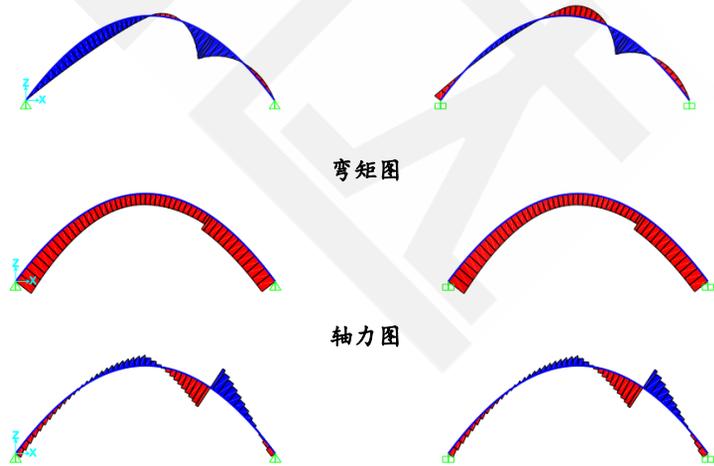
筑信达

## 弹性中心法

### ❖ 三铰拱 vs 无铰拱



$$y_s = \frac{\int y_1 \frac{ds}{EI}}{\int \frac{ds}{EI}}$$



弯矩图

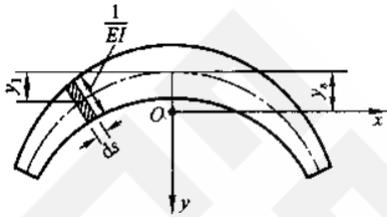
轴力图

剪力图

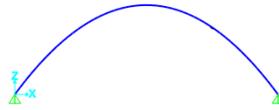
筑信达

## 弹性中心法

### 三铰拱 vs 无铰拱



$$y_s = \frac{\int y_1 \frac{ds}{EI}}{\int \frac{ds}{EI}}$$

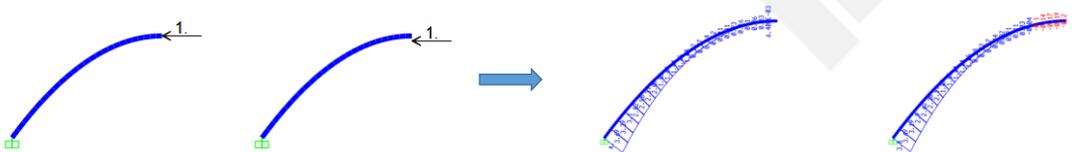
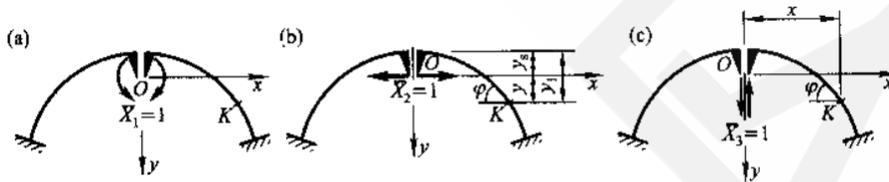


合理拱轴线

筑信达

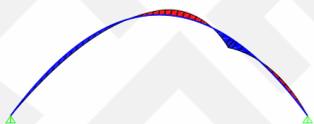
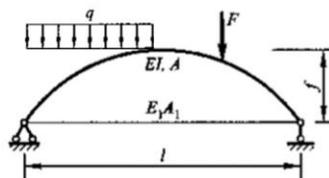
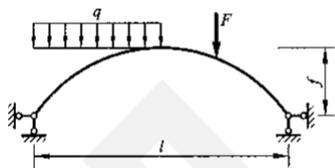
## 弹性中心法

### 刚臂

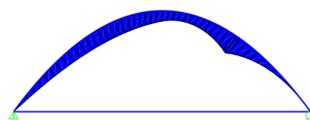


筑信达

## 两铰拱和系杆拱



有推力体系



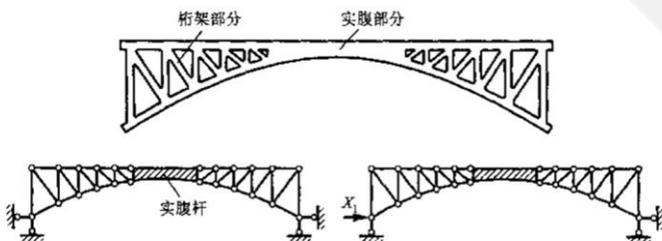
无推力体系

筑信达

## 桁架拱

### 两铰拱

- 桁架部分：杆，轴力为主
- 实腹部分：梁，轴力+弯矩+剪力



筑信达

# 谢 谢

**“The idea that an expert-system computer program, with artificial intelligence, will replace a **creative human** is an insult to all structural engineers.”**



加州大学伯克利分校终身名誉教授 Edward L. Wilson (著名的结构分析设计软件 SAP 的创始人) 在《Three Dimensional Static and Dynamic Analysis Of Structures》(<http://www.edwilson.org/Book/book.htm#Personal>) 一书中提到:

"Don't use a structural analysis program unless you fully understand the theory and approximations used within the program"

"Don't create a computer model until the loading, material properties and boundary conditions are clearly defined"